1 Definitions

$$v_{\mathrm{fd}} \equiv \text{velocity at full drift} \tag{1}$$

$$t \equiv \text{measured drift time} \tag{2}$$

$$t_0 \equiv \text{time zero} \tag{3}$$

$$t_{\mathrm{u}} \equiv \text{unknown offset } {\sim}1\mu\mathrm{s} \tag{4}$$

$$\mathrm{T} \equiv \text{real time} \tag{5}$$

$$v(\mathrm{T}) \equiv \text{time dependent velocity} \tag{6}$$

$$\text{assume } \langle v(\mathrm{T}) \rangle = v_{\mathrm{fd}}(\text{linear approximation}) \tag{7}$$

2 Equations

$$d_1 = v(\mathrm{T})(t_1 - t_0 - t_{\mathrm{u}}) \ , \ d_1 \equiv \text{distance to laser spot 1} \tag{8}$$

$$d_2 = v(\mathrm{T})(t_2 - t_0 - t_{\mathrm{u}}) \ , \ d_2 \equiv \text{distance to laser spot 2} \tag{9}$$

$$d_1 - d_2 = v(\mathrm{T})(t_1 - t_2) = v(\mathrm{T})\Delta t \equiv \Delta d \tag{10}$$

so,

$$v(\mathrm{T}) = \frac{\Delta d}{\Delta t} \tag{11}$$

$$\langle v(\mathrm{T}) \rangle = \Delta d \left\langle \frac{1}{\Delta t} \right\rangle \equiv \frac{\Delta d}{k} \ , \ k \equiv \left\langle \frac{1}{\Delta t} \right\rangle^{-1} \tag{12}$$

but **NOTE**:

$$\left\langle \frac{1}{\Delta t} \right\rangle \neq \frac{1}{\langle \Delta t \rangle} \tag{13}$$

so taking

$$\langle v(\mathrm{T})\rangle = \alpha v_{\mathrm{fd}}\ ,\ \text{ where } \alpha \sim 1 \text{ for now} \tag{14}$$

we have

$$\Delta d = \alpha v_{\mathrm{fd}} k = \alpha v_{\mathrm{fd}} \left\langle \frac{1}{\Delta t} \right\rangle^{-1} \tag{15}$$

and

$$v(\mathrm{T}) = \left(\frac{1}{\Delta t}\right) \alpha v_{\mathrm{fd}} k = \left(\frac{1}{\Delta t}\right) \alpha v_{\mathrm{fd}} \left\langle \frac{1}{\Delta t} \right\rangle^{-1} \tag{16}$$

giving finally,

$$d_1 = v(\mathrm{T})(t_1 - t_0 - t_{\mathrm{u}}) = \left(\frac{1}{\Delta t}\right) \alpha v_{\mathrm{fd}} \left\langle \frac{1}{\Delta t} \right\rangle^{-1} (t_1 - t_0 - t_{\mathrm{u}}) \tag{17}$$

and

$$d_2 = v(\mathrm{T})(t_2 - t_0 - t_{\mathrm{u}}) = \left(\frac{1}{\Delta t}\right) \alpha v_{\mathrm{fd}} \left\langle \frac{1}{\Delta t} \right\rangle^{-1} (t_2 - t_0 - t_{\mathrm{u}}) \tag{18}$$